Math 005C — Exam 1 Study Guide

This study guide is designed to help you prepare specifically for Exam 1. Every item below corresponds directly to skills or ideas that appear on the exam.

1. Vector Operations and Directed Magnitude

What you should be able to do:

- Find a vector between two points P and Q.
- Construct a vector with a specific magnitude in a given direction.

You should understand:

- How to find a unit vector (normalization).
- Scaling a unit vector to reach a target length.

2. Projections and Orthogonality

What you should be able to do:

- Compute the scalar projection ($\text{comp}_{\mathbf{b}}\ \mathbf{a}$).
- Compute the vector projection ($\text{proj}_{\mathbf{b}}\ \mathbf{a}$).

You should understand:

- The relationship between the dot product and the angle between vectors.
- How the scalar projection represents the signed magnitude of the "shadow" cast by one vector onto another.

3. Lines and Planes in 3D

What you should be able to do:

- Write parametric equations for a line passing through two specific points.
- Find the equation of a plane containing a point and a given line.
- Find the parametric equations for the line formed by the intersection of two planes.

You should understand:

- The role of the normal vector in defining a plane.
- How to use the cross product to find a direction vector orthogonal to two plane normals.

4. Vector-Valued Functions: Velocity and Tangents

What you should be able to do:

- Differentiate a vector-valued function $\mathbf{r}(t)$ to find velocity $\mathbf{v}(t)$.
- Calculate speed as the magnitude of velocity.
- Determine the parametric equations of a tangent line at a specific point.

You should understand:

- How to handle different function types (trig, log, exp) within a single vector.
- The geometric meaning of the tangent line as the linear approximation of a particle's path.

5. Acceleration Components and Curvature

What you should be able to do:

- Solve for the tangential component of acceleration (a_T).
- Compute the curvature kappa of a space curve at a given point.

You should understand:

- Why the tangential component of acceleration is the derivative of speed.
- How curvature represents the rate of change of the unit tangent vector with respect to arc length.

Practice Worksheet

1. Directed Magnitude

Find a vector of length 6 in the direction of the vector $\overrightarrow{PQ}$, where $P(1, -2, 3)$ and $Q(3, 2, -1)$.

2. Projections

Let $\mathbf{a} = \langle 2, -1, 5 \rangle$ and $\mathbf{b} = \langle 3, 4, 0 \rangle$.

a. Find the scalar projection of **a** onto **b**.

b. Find the vector projection of **a** onto **b**.

3. Lines in 3D

Find the parametric equations of the line that passes through the points $(-2, 5, 0)$ and $(1, 1, 4)$.

4. Planes and Lines

Find the equation of the plane through the point $P(4, 0, -2)$ containing the line $x = 1 + 3t, y = 2 - t, z = 4t$.

5. Intersection of Planes

Find parametric equations for the line of intersection of the planes
$x + 2y - z = 5$ and
$3x - y + 2z = 2$.

6. Particle Motion and Tangent Lines

Given $\mathbf{r}(t) = \langle e^{2t}, cos(\pi t), t^2 + 4 \rangle$.

a. Find the velocity of the particle in terms of t.

b. Find the speed of the particle in terms of t.

c. Find the parametric equations for the tangent line to the curve at $t = 0$.

7. Components of Acceleration

A particle moves with position function $\mathbf{r}(t) = \langle t, \ln t\,, t^2 \rangle$. Find the tangential component of the acceleration vector at $t = 1$.

8. Curvature

Find the curvature $\kappa(t)$ of the curve $\mathbf{r}(t) = \langle \sin 3t\,, \cos 3t\,, 4t \rangle$.